# THE DETERMINATION OF THE ZERO-SEQUENCE QUANTITIES OF POWER-TRANSMISSION LINES BY THE USE OF MODELS

BY

JAMES HENRY SMITH
E.E., University of Cincinnati, 1931
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AN ABSTRACT OF A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN
ENGINEERING IN THE GRADUATE COLLEGE OF
THE UNIVERSITY OF ILLINOIS,
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# THE DETERMINATION OF THE ZERO-SEQUENCE QUANTITIES OF POWER-TRANSMISSION LINES BY THE USE OF MODELS

## Introduction

While the algebraic expressions for the zero-sequence impedance quantities of a single three-phase power-transmission line are quite simple, the addition of one or more lines in parallel with the original one or the addition of one or more ground wires to the setup both cause the algebraic expressions for these impedance quantities to become quite complex, the degree of complexity increasing very rapidly as the number of lines and number of ground wires increases. Developments for a number of these expressions are given in the appendixes of the complete thesis report. In each case, it is necessary to make one or more simplifying assumptions pertaining to dimensional relations and to the distribution of the currents among the several conductors and over their cross-sectional areas, hence the equations do not represent precisely the true physical picture. From a practical point of view, however, experience in the power industry has shown that the inaccuracies these simplifying assumptions introduce into computed numerical values are negligible.

## The Thesis

The thesis is that if these zero-sequence impedance quantities can be obtained with accuracy acceptable to power-system engineers by actual measurement on a reduced-scale model of the transmission setup, then the costly and slightly inaccurate algebraic process can be eliminated. Furthermore, a model could be built to include many of the various mechanical irregularities in a transmission line or lines, which irregularities are quite often omitted or represented by what is thought to be an equivalent regular section of the line or lines when making algebraic computations, consequently the model could yield more accurate values of the impedance quantities than are computed. Publications referred to indicate that transmission-line constants are rarely known with less than five per-cent error, hence the requirement of accuracy imposed upon a model is not unreasonable.

## The Investigation

The investigation was concerned with finding out whether or not the zero-sequence impedance quantities of transmission lines can be determined with acceptable accuracy from reduced-scale models of the lines. Both analytical and experimental studies were made.

The analytical study shows how the Principle of Similitude can be used in the design of a reduced-scale model of a transmission-line setup in order that the zero-sequence impedance quantities can be obtained by measurements made on the model. The various inherent physical limitations imposed by the physical existence of only the commercially standard sizes of wire are considered in detail. Several examples illustrate the magnitudes of the errors introduced by off-scale diameters or areas of the conductors on the model and by off-scale values of the model's earth resistivity (Carson's rho). In general, a very complex state of affairs exists.

The experimental work consisted of setting up transmission-line models and then measuring the zero-sequence impedance quantities, after which the values measured were compared with the corresponding values computed from the theoretical equations that had been derived. Both single-conductor and three-phase transmission lines were studied, the three-phase lines having either delta or approximately vertical (tree-top) spacings. The three-phase lines were not transposed. Most of the experiments were performed at a principal frequency of 6 kc per second on 1/100th-scale models, and a few were performed with frequencies between 15 and 210 kc per second on several sizes of model. In all cases, the conductors on the model were of solid copper having a circular cross section. The "earth" in the models was either the earth itself, a salt-water trough, or a copper-wire counterpoise.

## Methods of Impedance Measurement

It was found that the zero-sequence capacitances could be measured satisfactorily in the audio range of frequency by means of a General Radio Company, Type 650-A Impedance Bridge. The zero-sequence resistances and inductances in this same frequency range were measured with sufficient accuracy by means of a simple alternating-current type of Wheatstone Bridge. The standard series-resonance method was employed to measure zero-sequence

inductive-impedance quantities at the frequencies between 15 and 210 kc per second, but the results of these high-frequency tests were not very satisfactory because the low values of Q of the impedances prevented the attainment of sufficiently sharp resonance.

## Results of the Investigation

The important results of the investigation are briefly as follows:

I. *For 1/100th-scale, 6-kc Models Built Over the Surface of the Actual Earth Itself:*

A. *For Lines Without Ground Wires:*

The zero-sequence resistance is unobtainable from a model because the 6-kc resistance of the earth does not obey the same formula used to compute the earth's resistance at 60 cycles per second. According to the 60-cycle formula, the 6-kc resistance of the earth is 9.528 ohms per mile. By actual measurement, this resistance was found to be approximately 82 ohms per mile.

2. The zero-sequence inductive reactance can be obtained with about 2 per-cent error from a model, provided:

a) Wire can be obtained having exactly, or at least approximately, 1/100th of the full-size GMR (geometric mean radius). For ordinary cases, the least off-scale wire commercially procurable would introduce less than 0.5 per-cent error in the reactance.

b) The model happens to exhibit a value of Carson's rho equal to 1/100th or thereabouts, say within plus or minus 10 per cent, of the value in the full-size setup. The possibility of this happening is so remote as to preclude the practical use of a model. Values of rho equal to 0.292 and 0.500 meter-ohm were obtained on the models studied. In most practical work, a value of 1.00 meter-ohm would be required of the model.

3. The zero-sequence capacitance can be obtained with about 1 per-cent error from a model, provided wire can be obtained having exactly, or at least approxi-

mately, 1/100th of the effective outside diameter of the full-size conductor. For ordinary cases, the least off-scale wire commercially procurable would introduce less than 0.5 per-cent error in the capacitance.

4. A size of wire favoring the determination of the zero-sequence inductive reactance would cause, at the most, a 1.5 per-cent error in the zero-sequence capacitance as determined from such a model, and vice versa.
5. In determining zero-sequence capacitance, major irregularities in the contour of the earth's surface and sag in the conductors on the full-size setup and on the model cannot be neglected, for this capacitance is a function of the conductor height. Effective heights of the conductors must be used. In the cases of the lines studied, the effective capacitive height of a conductor was approximately 72 per cent of the height at the supports. More research needs to be done on the matter of the effective capacitive height of a line.

B. *For Lines With Ground Wires:*

1. The zero-sequence resistance and inductive reactance cannot be obtained from a model. There are several reasons for this. First, the formula that is used to compute the earth's resistance at power frequencies cannot be used at audio frequencies. Second, the rod-to-earth resistances of the two ground rods needed to make contact with the earth cannot be directly subtracted from the measured value of the model's resistance. Third, in this case the zero-sequence resistance and reactance are interdependent, so that the much-too-high value of earth resistance plus the 15-ohm rod-to-earth resistance of each ground rod cause the measured inductance to be too small.
2. The zero-sequence capacitance of one line can be determined with about 1 per-cent error from a model, provided it is possible to procure wire having exactly, or at least approximately, 1/100th of the effective outside diameter of the full-size conductor, and provided the effective heights, rather than actual heights, of the conductors on the model and full-size line are used.

3. The zero-sequence capacitance of two or more paralleled lines is unobtainable from a model. The reasons for this are not known definitely, but no doubt the incorrect values of the zero-sequence resistance and inductance obtaining on the model cause an incorrect distribution of the charging currents and, hence, an incorrect value of zero-sequence capacitance is measured. Of course, this situation would exist with only one line, but not to as great an extent.

The conductors were not transposed on any of the models built. The theoretical equations for all impedance quantities are based on perfect transpositions. This neglect of transpositions on the models may be a factor causing some error in all the impedance values obtained from the models. It is not believed, however, that much error was introduced by the neglect of transpositions.

II. *For 1/100th-scale, 6-kc Models Built Over a Salt-Water Trough.*

A. *For the Determination of Zero-Sequence Resistance and Inductive Reactance:*

In principle, this type of model acts like one built over the earth itself, but it has features which eliminate the inherent difficulties of that type of model, thus:

1. The troublesome ground rods are eliminated.
2. Further research should develop means for obtaining simultaneously the desired value of Carson's rho and of "earth" resistance.

B. *For the Determination of Zero-Sequence Capacitance*

The model is unsuited to this determination, for the capacitances obtained are too small to be measured accurately.

III. *For 1/100th-scale, 6-kc Models Built Over a Counterpoise Made of Copper Wires*

A. *For the Determination of Zero-Sequence Resistance and Inductive Reactance:*

In principle, this type of model acts somewhat like one built over the earth itself, but it has features which eliminate the inherent difficulties of that type of model, thus:

1. The troublesome ground rods are eliminated.
2. The model can be built indoors.
3. The model can be built simply and cheaply. In these respects, this type of model surpasses the water-trough type of model.
4. Further research should develop a counterpoise design for obtaining simultaneously the desired value of Carson's rho and of "earth" resistance.
5. With a counterpoise made of 24, paralleled, No. 26, solid copper wires, spaced horizontally on 2-inch centers, the two impedance quantities were obtained with about 5 per-cent error in each when the ground wires were not in use, and with about 6 per-cent error in each when the ground wires were in use. Hence, the counterpoise offers a likely solution to the entire problem except, of course, for the difficulty of finding line conductors which would satisfy at once all three parts of the Principle of Similitude.

B. *For the Determination of Zero-Sequence Capacitance:* The single-layer counterpoise described above resulted in values of capacitances having, on the average, about 40 per-cent error. The per-cent errors ranged from zero to 83.9 per cent. Possibly a multilayer counterpoise would yield better results. More research is required.

IV. *Length of 1/100th-scale Model*

The model must be at least 200 feet long, and a length of 200 to 325 feet is recommended.

V. *Carson's Rho on a 1/100th-scale Model Operated at Audio Frequencies*

In the 1-12 kc range of frequency, Carson's rho for a 1/100th-scale model is a complicated function of frequency, line height, line configuration, and the value of Davy's rho (ordinary resistivity) for the material making up the model's "earth." If this "earth" is a counterpoise, then the arrangement of the wires on the counterpoise is an additional factor in determining the value of Carson's rho.

VI. *Line Height on a 1/100th-scale Model*

It was found experimentally that for the heights which would normally be encountered in modeling a transmission setup to 1/100th scale, the resistance and inductive reactance of a single conductor with an earth return circuit were independent of the height of the conductor above the earth. Heights of 2 to 16 inches, inclusive, were used in the study. The results verified the corresponding theory.

VII. *Models Designed to Operate at Frequencies Above the Audio Range*

For a 60-cycle power frequency, the Principle of Similitude requires frequencies of either 60,000 or 600,000 cycles per second, or higher values similarly involving the factor 60, and the model has to be built correspondingly to either 1/1000th or 1/10,000th scale, or smaller. As a result, the following difficulties exist:

A. The model is so small that the physical distances cannot be attained easily and with sufficient accuracy.

B. The diameters of wire required on the model would, in many cases, be smaller than that of No. 40, which is the smallest diameter wire manufactured.

C. The zero-sequence resistance and inductive reactance at 60 kc have to be measured by establishing series resonance in the circuit. The Q of the circuit is so small that sharp resonance cannot be obtained, and hence the values obtained for the impedance quantities are quite inaccurate.

D. At 600 kc, radiation effects are likely to complicate matters an objectionable amount.

## General Conclusions

Once a material is found, out of which a wire could be made that would favor the determination of both zero-sequence resistance and inductive reactance from a model in accordance with the Principle of Similitude, then the counterpoise type of earth or water-trough type of earth, further developed as suggested in the body of the report, would complete a 6-kc setup that would be satisfactory for determining all three zero-sequence impedance quantities.

With the kind of conductors available today, the determination

of both zero-sequence resistance and inductive reactance from a model built over either of these two types of simulated earth is possible in only those cases where the inductive reactance quantity is entirely or practically independent of the off-scale value of the resistance of the model's conductors, or where it just so happened, by chance, that a size of wire existed that satisfied simultaneously the required relations for zero-sequence resistance and inductive reactance.

For the zero-sequence capacitance to be determined accurately, the diameters of the conductors on the model have to be reasonably close to 1/100th those on the actual setup, the number 100 being the recommended scale-reduction factor.

# VITA

James Henry Smith was born on January 24, 1908, in Covington, Kentucky. He attended the Sixth District School in Covington, completing the sixth grade in June 1919, at which time the family removed to Cincinnati, Ohio. The seventh and eighth grades were completed in June 1921 at Westwood Public School in Cincinnati. In June 1925, he graduated from the Industrial Arts Course taken at Hughes High School in Cincinnati.

He continued his education at the University of Cincinnati, taking the special six-year, co-operative engineering course leading to the degrees of Electrical Engineer and Master of Science in Engineering, which degrees were received on June 13, 1931. The title of the thesis serving for both degrees was "A Self-Regulating Transformer." During the school years 1936-37 and 1937-38 he attended the University of Cincinnati Evening School, studying in the field of chemistry. The first half of the summer of 1939 was spent in studying German at this same institution.

While an Instructor in Electrical Engineering at the University of Illinois, September 1939 to July 1941, he did further undergraduate and graduate work in his free time and during the intervening and succeeding summers, and during the fall of 1941 and spring of 1946, resulting in A.B. and B.S. degrees on June 16, 1946. The A.B. was in philosophy, French, and German; the B.S. was in chemistry and mathematics.

Later, while on the teaching staff at Cornell University, he attended as an auditor a course in educational psychology and advanced courses in communication and high-voltage engineering.

As a co-operative student at Cincinnati he obtained the following practical experience: tool-maker apprentice and, later, assistant to the chief plant and test engineer at The Lunkenheimer Co., Cincinnati; laboratory-maintenance assistant in the Department of Electrical Engineering at the University of Cincinnati; mechanical-designer and assistant to the chief mechanical engineer at The Continental Can Co., Norwood, Ohio; research assistant in the Basic Science Research Laboratory at the University of Cincinnati; and, finally, test electrician at The Cincinnati Gas & Electric Company, Cincinnati, Ohio.

From June 16, 1931, to July 27, 1936, the author worked as an apprentice electrical engineer for The Cincinnati Gas & Electric Company. Here experience was gained in practically all phases of electric power-system maintenance, testing, and operation. Near the end of this period experience was gained in electrical drafting. He became progressively a layout draftsman and checker, and, on July 27, 1936, a design engineer specializing in switchboards, control, metering, and protective relaying. In absences of the regular supervisor over the design-engineering section, the author served in

his place. The author remained a design engineer here until September 15, 1938, at which time he resigned to enter the teaching profession. He returned, however, during the last half of the summer of 1939, working out the remainder of the summer vacation after having spent the first half of this summer studying German at the University of Cincinnati.

During the school year 1938-39 he was Professor of Electrical Engineering and Head of that department at the Detroit Institute of Technology, Detroit, Michigan. He resigned in order to take a position where graduate work might be done while teaching.

During the school years 1939-41 we was an Instructor in Electrical Engineering at the University of Illinois. The fall term of 1941-42 and the summer terms of 1946 and 1947 were spent as a full-time graduate student at this same institution completing, for the most part, all formal course requirements.

From February 2, 1942, to September 1, 1942, the author was an Instructor in Electrical Engineering at Cornell University, Ithaca, New York, and on September 1, 1942, he became an Assistant Professor. He remained in this capacity until June 30, 1947. During World War II he was also a member of the teaching staff of the U. S. Naval Training School established at Cornell for officers and non-commissioned men.

On September 1, 1947, the author became an Associate Professor of Electrical Engineering at The University of Nebraska, Lincoln, Nebraska, and is a member of the staff at the present time.

He is a Member of the American Institute of Electrical Engineers and of The American Society for Engineering Education. He has a professional engineer's license in electrical engineering, No. 4638, in the State of Ohio.